Using Art in Teaching Philosophy of Mathematics and why it has Nothing and Everything to do with Democracy

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Introduction
Teaching philosophy of mathematics is a challenge. It is even more of a challenge to do it in a meaningful way, where the different philosophies become more than texts to be learned, and are confronted with the students’ existing perceptions of mathematics. Last year, I decided to bring in art activities derived from the Bridging Polarities art course. The students expressed their enjoyment with the approach, and it did appear to have some positive effects on their learning, though not in every case. In this paper, I describe my very personal experience of the course and relate it to the overall purpose of teaching the philosophy of mathematics.

Colouring Mathematics
The class is small, only twelve students are registered for this Masters course and a few are not attending today. I can hear them talking as they walk away to find a spot to sit or a place to pick up something to eat during the break. Then I get busy. The small lecture theatre I have been assigned has fixed chairs and swivel tables, but luckily there are two old desks available. They are heavy wooden giants and I have to put all of my 64 kg behind pushing them into position. Then I stick A3 size art paper on the desks with masking tape and put out pastels, three scalpels and some wet cloths.

The students are open to the task, except one who is visibly pulling back from the activity. I ask them to scrape white pastel onto the paper using the scalpels. Then they must rub the white powder into the paper with their hands. This is to prepare the paper so that the colour they add next is not so easily absorbed into the surface. The students carry out the task with what appears to be enthusiasm[1], all ending up with white fingertips in the process.

Next, I ask them to close their eyes and imagine first one, then another aspect of mathematics. They must then choose a colour to signify each aspect. On one half of the paper, they must put the one colour; on the other half of the paper, the other colour. Then I ask them to see what happens when the two colours meet and begin to merge.[2] As they work, I notice how the Zulu students and students of Indian origin tend to stay with others
of the same ‘race’ group. The colours merge on the paper, but they never merge completely in the colours of the lecture theatre.

The concentration is thick in the room. Only the one reluctant student finishes quickly. His picture is a blur of brown, as he has just rubbed the two colours together. When he looks around at the beautiful pictures emerging under the hands of the other students, I am not sure if he is critical of the activity or himself.

After they have all completed their art work, I ask the students to share their experiences, their feelings, what aspects of mathematics they were giving colour to, and to look around at each other’s work.

The differences in choice of colour are striking, as are the different aspects of mathematics on which they have chosen to focus. It is the first manifestation that we experience mathematics differently, that there is a personal and experiential reality to this subject, so often taught as merely a set of rules and/or a logical and coherent system.

Some of the students have primarily enjoyed the experience of the artwork, the presence it brought about to be so engaged in this kind of activity; others express realisations about mathematics as having several coexisting aspects. There is a sense of excitement and joy with most of the students as they proudly take their pictures and leave the class.

**Teaching the Philosophy of Mathematics - What?**

When I was first asked to teach the philosophy of mathematics, I was a young PhD student in Denmark. Together with a course in mathematics education, the course in philosophy and history of mathematics and science was part of a 5-year programme leading to a Masters degree with mathematics as a major. I had no knowledge of what the philosophy of mathematics was or why it could be relevant. I taught the course while being only a few chapters ahead of the students in the textbook we used (Skovsmose, 1990).

As I taught the course repeatedly over the years to follow, experimenting with new readings and new approaches, I became increasingly intrigued by the content as well as the impact it had on the students. Here were students who had chosen mathematics as their major and had already completed a few years of study, yet engaging with the philosophy of mathematics changed the way in which they connected with the content in their mathematics courses. This was even more evident to the lecturers of mathematics than it was to the students themselves, if I am to trust the feedback I got from these lecturers. They would come to me and indicate how the students had started to take more responsibility for their learning, asked questions about the types of proof, engaged with the origins of mathematical concepts, etc.

In terms of mathematics education, it was Sfard’s famous paper on the dual nature of mathematical concepts (Sfard, 1991) that made the difference, and made the students look differently at the notions they encountered in their courses. In the philosophy course, it was having their understanding of what mathematics is shaken up repeatedly.

Years later, I find myself teaching about the philosophies of mathematics again, but this time as part of a one year postgraduate Masters programme in mathematics education at University of KwaZulu-Natal in South Africa. The programme is different, the students are older and of more diverse background, but the philosophies we work with are the same. [3]
This is not the place to go into great detail about the philosophies of mathematics, only to touch on the main points. One important progression is the change of a focus on ‘programmes’ for mathematics, (in a sense, normative philosophies), to descriptions of what mathematics is.

One of the earliest programmes we address in the course is logicism. I describe the logicist programme in simple terms as:

**Logicism’s programme**

If every provable statement is true, then the system of axioms is healthy.

If every truth statement can be proven, then the axiomatic system is complete.

The goal is to have a healthy and complete system.

There would be two ways of doing this: one could prove in some fancy way that such a system is possible; or one could simply try to construct such a system. The logicist programme is to do the latter.

It all seems fairly reasonable, until we start looking at how this programme breaks down: the paradoxes possible when working with concept extensions and the counterintuitive attempt to repair the system; the weakness of the system as it cannot give us elements that we consider part of mathematics; the incompleteness of the system - that there will always be statements that we cannot prove true or false; the quarrels about the postulates regarding infinite sets. Of course, more can be said on philosophical grounds (see Körner, 1960).

Next, we touch on Hilbert’s meta-mathematics, which offers to solve the problems of logicism by proving that a healthy and complete system is possible. Instead of talking of truth, Hilbert talks of consistency. The requirement is not only that a formal system must formalize a consistent theory, but also that it formalizes what it aims to formalize completely.

In comes again Gödel’s incompleteness theorems:

- A formal system that only formalizes elementary arithmetic will not formalize it completely. There will always be statements, the truth of which cannot be decided within the system - no matter how much we expand that system.
- If F is a consistent system, and f is a formalization of the statement “F is consistent,” then f is not a formal theorem in F. So the consistency of a formal system cannot be shown within a system itself.

And from there, we move towards more descriptive philosophies: conventionalism, Lakatos’ meta-mathematics, socio-constructivism, …

When I first taught this, I took the students through the main idea in Gödel’s work. As the students I teach now are not majoring in mathematics but in mathematics education, this would probably make the main point drown in symbols and deductions, so I simply tell them the results.
But that’s not the only difference. When I first taught philosophies of mathematics, the students reacted to Gödel’s incompleteness theorems with shock and denial. They had a clear conception of mathematics, which they had to throw overboard during the course, and that was an emotional and intellectual adjustment. It was an important one I thought, so when I started teaching philosophy of mathematics again last year to the mathematics education Masters students, I organized the course with the intention of moving towards a similar classroom experience. But without going into Gödel’s theorems, what would lead the students to a similarly emotional confrontation with their existing conceptions?

**Conversations in Clay**

Words are useful. Sometimes, they also get in the way. They get in the way of true communication, connection on a deeper plane. How deep one can go through a deceptively simple non-verbal activity, I discovered when Andrea Raath asked her art students to do a conversation in clay.

With a lump of clay the size of my fist, I expressed the many emotions I was struggling with at the time. It was a turbulent time in my life, and a lot went into that clay. Had you just looked at the clay, you may very well not have picked that up; it ended up as a smoothened ball with a compression on the one side. I was then to hand the clay over to another art student. He carefully caressed the clay, working with the shape it had taken from my hands, extending it carefully. And so the clay went back and forth, and in the end I felt attended to and seen and cared for more than I remembered having felt in a long, long time.

Andrea told me how emotionally disturbing it had been to some of her students when another student was not sensitive enough to pick up the emotions put into the clay. The confrontation with the ways in which the shapes were not extended but altered drastically was hurtful.

I was wondering if this could be given an intellectual parallel. So I asked the students in my Masters class to work in pairs. One was to put into the clay how s/he felt when thinking about mathematics as a consistent, healthy and complete system of truths. Then the other would take over the clay, and impress into the clay the impact of Gödel’s work.

I was excited.

So were the students. They took to the clay eagerly.

Then, they started making little figurines or mathematical shapes: triangles, pyramids, etc. I repeated the instructions. It had little effect. They did not express their emotional relationship to mathematics or their understanding of mathematics, but tried to impress into the clay images from mathematics. And passing the clay on to see it altered had no emotional impact - obvious, since emotions had not been expressed.

My intentions for the activity were caught by the students’ past experiences with clay as something you use to make models of objects. However, there was more to it than that. The students clearly expressed that they had no emotions related to the death of the two big programmes, logicism and Hilbert’s meta-mathematics. A sense of relativistic truth was conveyed, and why should that be different for mathematics?

I was surprised. I still am somewhat puzzled as to how these students could feel this way,
given their past exposure to mathematics. It was an anticlimax. “So, logicism is out, Hilbert’s meta-mathematics is out. Okay, what’s next?” It made me doubt the purpose in teaching this course in the first place. But I will return to that shortly.

**Teaching the Philosophy of Mathematics - How?**

When I taught the course on the philosophy of mathematics in Denmark, it spanned many weeks. We had time to build up an entire philosophy one morning and tear it down the next week. But the format I followed was pretty much the same throughout: give an overview, expect the students to read a text, let them work in groups on unpacking the text, summarising, and then moving on to the next topic.

This is unlike how I teach mathematics content courses or mathematics education courses. Like most people in mathematics education, I try to make the courses engaging, practical, related to the students’ experiential reality, and so forth - while recognising that this in itself constitutes a particular ideology about learning and relevance, of which I am also critical. The challenge was to do the same for the philosophy course. But the only experiences we have to relate to are the students’ experiences of mathematics as a subject in school and teacher education - none of them are mathematicians. To circumvent this, I used a historical text, which obviously does not replace personal experiences, but does at least provide a link to these by unpacking what went into the development of - in this case - the mathematics of prime numbers (du Sautoy, 2003).

This text provides narratives about the mathematicians who contributed to prime number theory over the centuries, but also offers something in terms of political and cultural context that indicates that individual mathematicians are social as well. We supplemented this by reading a text that explains a major shift in the practice of mathematics by the change in the economical circumstances of mathematicians - when they had to supplement their income by teaching, the details of previously established proof and theorems had to be unpacked (Grabiner, 1974/1986). Finally, we read excerpts from Lakatos’ Proofs and Refutations (Lakatos, 1976) to focus on what mathematicians do - a descriptive account.

But it was still so utterly ‘in the head’. Not only in that it was intellectual, but also in that it did not allow for any embodied relations to the knowledge. This is why I decided to include the art components. What else would make this more tangible to the students? I still do not know …

**Perspective Pluralism through Colour**

After engaging with the more descriptive philosophies of mathematics, the history of mathematics - looking both at individuals and at the larger socio-political context, having examined the products of mathematics as well as the practices of mathematicians … I asked them to make another picture in the same way as before.

My own version is shown here:
The result was, according to the sentiments expressed by the students afterwards, an integrated ‘sense’ of what mathematics is; a coming-together of the different aspects of mathematics; a tacit understanding that it is in the meeting of perspectives that we begin to get a sense of the complexity of the phenomenon we refer to as ‘mathematics’.

Though I did not introduce the students to the terminology, this is what Ole Skovsmose (Skovsmose, 1990) referred to as ‘perspective pluralism’ - the idea that we can only comprehend what mathematics is when we move through a range of perspectives on it: from the objects of mathematics; via the processes and practices of mathematics; to the applications, uses and functions of mathematics.

But this obviously has nothing to do with the last word in my title, democracy. It’s an intellectual endeavour for its own sake. Clearly … Or?

**Perspective Pluralism and Democracy**

Some years ago, a South African colleague spent some time in Denmark. Later, she shared a narrative with me. As part of her visit, she was invited to attend a community meeting. The topic of the meeting was not one that made me proud; some members of the community were concerned that the high inflow of foreigners to the area had a negative impact on their children’s schooling. According to my colleague, there were obvious racist or xenophobic undertones to some expressions of this concern.

The tone of the meeting, however, was what had made the greatest impression on my colleague. The community members had listened to each other’s views and arguments in a relatively calm atmosphere. The meeting was mostly dominated by the aim of coming to understand the perspective of others, reflecting the deep knowledge that only by bringing all the elements together can we get a fuller picture of the situation and choose the most suitable solution.

To me, the true sense of democracy.

A majority is not the people. For the people to rule, each of us must live with our truth being only one perspective on the whole, with dialogue, with the responsibilities that come from giving others the same rights that we demand for ourselves. It is a responsibility to stay informed, to consider the viewpoints of others, to engage in perspective pluralism.

Do the students’ experiences with perspective pluralism influence their way of engaging with the perspective of others with the aim of first seeking to understand before trying to argue one’s own point or reach consensus? Do they come to see that there are times when consensus is not desirable or possible, and a pluralism of perspectives must be allowed to coexist?

Do the students’ experiences with bringing two colours together to see how they both merge to unfold new hues and remain in contrast, creating a beautiful and emotionally captivating picture, influence their ability to accept and hold situations where differences of opinions and perspectives manifest?

Do the students’ experiences with how differently they all viewed mathematics, despite having similar educational experiences and all working as teachers within the same national curriculum guidelines direct them to see the beauty and usefulness in not all of us perceiving things the same way?

Did we contribute to the rainbow nation[4]?

I do not know. I only hope.

So perhaps this has nothing to do with democracy. Perhaps it has everything to do with it.

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[1] As mentioned, this is a very subjective experience of the events. To me, the students appeared enthusiastic. They later provided feedback on the course, indicating that they had enjoyed the art activities. But when I write that ‘the concentration is thick’ etc., I acknowledge that I am expressing my subjective experience of the event and the relations between the students and the task.

[2] It is only fair to acknowledge that I have developed this activity from my own experiences in an art course called Bridging Polarities taught by Andrea Raath in Cape Town. The same applies to the ‘Conversations in Clay’ technique described later. While Andrea has immense expertise in using art to further self-exploration, I have none. I shamelessly used the techniques to explore ‘polarities’ of mathematics, in order to foster the intellectual engagement with mathematics and its philosophies.

[3] Ideally, I should have contemplated if I could have brought in other views on mathematics, perhaps tried to foreground African thoughts more. But there was no time; the teaching load at higher education institutions in Africa is often such that there is insufficient time for deeper engagement with the curriculum.

[4] The widely adapted term used by the two most well-known political icons of the new South Africa, Desmond Tutu and Nelson Mandela, to refer to a nation built around celebrating and respecting diversity.

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References


**About the Author**

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