PROLOGUE: A Parable
In set theory, one can always take the “power set” of a set, and this new “power set” always has greater cardinality than the original set. (The power set is the set of all subsets of the original set. Greater cardinality is another way of saying that a set is “bigger.”) The explanation of this idea is quite simple once one is comfortable with the words of set theory. The idea is to imagine that it is possible to create a one-to-one correspondence between the original set and its power set, because, as Cantor showed, such a correspondence would mean that the two sets have the same cardinality[1].
Cantor’s clever idea was that we construct a new subset of the original set like this: for each element of the original set, we consider the correspondence we are assuming exists, and if that element is paired with a set that contains that element, then we will not put that element into the set we are constructing. Otherwise, if the element is paired with a subset that does not contain that element, then we do put that element into the set we are constructing. Sounds silly, but it is all we need to show that this new subset could never have been included in the one-to-one correspondence!

So, by assuming we have a one-to-one correspondence, we demonstrate that we do not have a one-to-one correspondence. We have no other choice than to accept that our assumption must have been wrong. There just couldn’t be such a correspondence, and in fact, there are things in the power set that are not matched with anything in the original set, no matter what matching we could concoct. So we can “see” that the power set has leftovers, meaning it is “bigger” and has more things in it than the original set.

**Why was that parable the opening of this article?**

First of all, note the kind of application of reasoning, which is designed to compel us now to believe something. This type of rationality tends to be the underlying ideology of any research project. The odd genre of proof by contradiction, *reductio ad absurdum* (i.e., where someone says, suppose what I really want to argue against is true, just so I can demonstrate a fundamental contradiction with another thing you believe, and therefore convince you that some thing you might believe is undeniably inconsistent with some other things that you believe) is one of a family of practices—or, in the language to follow, one form of a structuration of social practices, or one kind of process of constraining and enabling possible forms of life and ways of being in the world.

There is a kind of oddity in this rationality: it is all a fantasy, given that we beg our audience to enjoy a good story, simply in order to lead them to dismiss the main points of the story by the end of the tale. The art of storytelling becomes essential to the ways of working as a mathematician. In the collage of stories that make up much of this article, I ask you also to consider that there are parallel tales of educational researchers unfolding in a never-ending process more commonly known as the meta-narrative of social science. And I ask you to further consider this collage together as an analogous parable with an analogous ‘moral.’ Perhaps it is the case that educational research is one big exercise in proof by contradiction?[2] If so, we may find solace in alternative practices where the purpose is something other than the construction of such parables. Would there be alternative purposes, then? If so, what would or could these alternative purposes be? Could research be grounded in something other than purpose?

Note, too, that such chains of deductions and inferences that are found in (educational) research are pretty much the necessary elements that drive the plot and character development of any detective story. It ‘pops out’ that detective stories and educational research might have something to learn from each other. Researchers and mathematicians work as if it is already decided that they should or must concoct illustrations of what they have experienced and are imagining. The illustrations represent generalized concepts and points of argument, whether by analogy, or metaphor, and in the process, they conjure up an architecture of practice.

Two features of the mathematics education architecture that seem to me to be particularly relevant are the support of **sense-making** by pupils, and the overarching aim of facility...
with representations. By ‘sense-making,’ I am referring to the common assumption that our task as mathematics educators is to help young people to make sense of mathematics. We receive mathematics as a reasonable and logical world within which one should be made to feel comfortable and secure. We often agree that mathematics is the one place where we can be certain about what we know and whether or not we are correct. My comments have to do with the power of these assumptions to enable specific kinds of educational experiences while also perhaps failing to allow pupils to fully appreciate the wonders and powers of mathematical modes of inquiry and understanding.

With ‘representations,’ I ask us to consider also the power of our typical pedagogies, which tend to lead students from the concrete to the abstract, and also to move students away from specific instances of mathematics in the world toward general representations of these instances. We might think of the representations (or of the ideas represented) as the actual material and content of the mathematics itself. I mean here simple things like: numerals to represent numbers of things; drawings of shapes to represent ideal geometric relationships; fractions to represent parts of wholes, proportions, and ratios; equations to represent functional relationships, letters to represent variables that may take on different values, and so on. Other representations model mathematical concepts and relationships, such as base-ten blocks for arithmetic operations, drawings of rectangles or circles for fractions and ratios, or graphs which visually represent algebraic equations.

In my experience, much of mathematics education aims to help students to develop artistic virtuosity with mathematical representations for communicating their ideas. However, if we take this artistic virtuosity seriously, then critics of artistic practice sometimes suggest that representation is not always the aim of art, and in fact, representation often violates art itself. What would young children as mathematical artists do, then, if they would not primarily be practicing forms of mathematical representation? I will return to this question, because it is connected most directly with broad, social contexts and the role of mathematics education in democratic practices, even as we focus on the individual mathematicians in our kindergartens, primary school classrooms, and on the adolescent mathematicians with whom we work day-to-day.

For now, as a conclusion to this introduction, we can consider a further morality lesson from the parable of Cantor and infinite sets: Cantor himself spent most of the last years of his life working to prove the well-known continuum hypothesis, which states that there are no infinities between a countably infinite set and its power set. He was also in and out of asylums, as he first proved his hypothesis true, then perhaps proved it false, then true, and so on. Only years later do we have Kurt Gödel and Paul Cohen to thank for a somewhat disturbing (or should we say delightful?) conclusion to the mystery of the continuum hypothesis. They used systems of deduction and rational certainty that lead to the belief that we can never know whether the continuum hypothesis is true or false. We can’t and never will be able to prove it is true, and we can’t and never will be able to prove that it is false. Similarly, in postmodern detective fiction, we often are left at the end without knowing for sure what really happened. We might even be wondering if what we thought happened

http://ccfi.educ.ubc.ca/publication/insights/v13n01/articles/appelbaum/index.html
could have happened at all. And the crisis of educational research is that no data sets or rational arguments are enough to convince any one of the “one best system” (Tyack 1974) … in fact, we are more often led to the conclusion (if that is the best word to use) that there is no best system, no right choice, and so we can only continue to participate in the world of mathematics if such certainty is no longer what we strive for. In educational research and/or detective fiction, it might also be conceivable that a narrative that leads to certain conclusions might be left behind as we enter whole new worlds of practice.

In reference to the parable, it is also the case that most mathematicians these days have a working belief upon which they act with regard to the continuum hypothesis. They tend to suspect that there are indeed infinities between a set and its power set. Infinities tend to look a lot like our ordered and friendly counting numbers, and we know there are fractions between the counting numbers. Many of us want to believe there are fractional infinities. Is this nothing more than John Dewey’s (1929) definition of “belief,” from his Pedagogic Creed? Dewey stressed “those things upon which we are prepared to act,” as opposed to the beliefs we tell ourselves and others about. That is, the “real beliefs” that may or may not coincide with the tales we tell, the narratives of self-disclosure, self-revelation, public proclamation, and so on. In educational research, we may not be grounded in certainty, but we may also work in ways that re-inscribe our beliefs. We may participate in this work in ways that reconstruct our unspoken beliefs upon which we act, not as invisible ideology but as commitments; the beliefs are encoded in and by the illustrations of what we know. How might we juxtapose our narratives of what we do with those narratives of what we believe we are doing? Is it possible that our tales of research might be as useful or more useful as forms of undetective ‘unconclusions,’ destabilizing the expectation of purpose and findings?

My research story is a multiple tale. It is a narrative, along with the parallel, companion, reflexive narrative of what it has meant to come to be a narrative; or more plainly, it is multiple tales, of a particular educational project. It is informed by postmodern detective fiction. My collage, my work of textual art, is inspired by Patti Lather’s (2007) model of refracting upon the process of post-research as the process is unfolding upon itself. I am sharing the un-questions of an anti-curriculum project, yearning for a ‘way’ to avoid searching for the logic imposed upon the process by the guilty party. It is also an homage to a lovely little fictionalized memoir, Pasta all’ infinito. Meine italienische Reise in die Mathematik (Pasta all’ Infinito: My Italian Journey into Mathematics), by Albrecht Beutelspacher (2001), a highly readable story that holds our avid attention through such rhetorical devices as having characters discuss the details and implications of oft-told mathematical parables. So my research is simultaneously a process of interpretation in action, the writing of a parable of experience, and the reading of detective fiction. As a work of reading and writing experience, it is an effort of both reading and writing curriculum theory (Appelbaum 2008a).

What I mean by this is I worked as a reader, interpreting fictional, mathematical literature as educational theory; this might be named reading curriculum theory. At the same time, I worked to invent a discourse that could communicate the context and processes of curriculum work, which might be named writing curriculum theory. However, neither form of working was distinct and separable from the other: while one might say I was reading the literature as curriculum theory, I was also reading this literature with other purposes in mind—pleasure, entertainment, evaluating possible class materials, etc. The curriculum theory work might be otherwise named as political intervention, relationship building, friendship, and so on. Because this kind of work is both reading and writing, but also at the same time neither purely reading nor simply writing, the experience is one of a Deluzian
(Deleuze and Guattari 1991; Braidotti 1994) nomadic epistemology. I found myself inventing new concepts that coexist with the limit and the resource, the bounded and the unbounded, the constraining and enabling power sets of structurations; yet I was acting, interacting, mediating, and interrogating those dualistic binaries. In this work, as in *Pasta al Infinito*, the unresolved and unknown are characters as prominent as conclusions and resolutions. Most broadly, however, it is through the artistic practices of pastiche and collage that I have been able to express this work for myself, and through which I share it with you.

**The Undetective enters the Project**

Noel Gough writes:

> At least three kinds of intertextual continuities … link stories of educational inquiry with detective fiction … First … reports of educational research resemble detective stories insofar as they narrate quests to determine “the truth” about something that is problematic or puzzling—stories in which “investigators” seek … “to reveal, disclose, to know, and … to empower.”

Second, following Eco’s … characterization of the novel as “a machine for generating interpretations”… consider the extent to which the characteristic ways in which detective stories generate interpretations [and how they] resemble the textual “machineries” used in the discursive production of educational research.

Third, … consider some of the ways in which recent transformations of both detective fiction and educational inquiry can be understood as comparable — and intertextually linked — manifestations of cultural and semiotic shifts signified by various notions of postmodernism and postmodernity.

—Gough 2007: 61

Gough critiques research as detection, but finds solace in postmodern “undetectives” whose mysteries are not quite solved in the sense of a resolution grounded in truth; these stories are engrossing in ways that lead to new, possibly postmodern, forms of satisfaction, pleasure, knowledge, and transformation.

I was drawn to Gough’s article at a particular moment in my own professional work. The project asked secondary students and politicians in Potsdam, Germany, to meet and discuss the role of mathematics in democracy during that nation’s official *Jahr der Mathematik* (‘year of mathematics’)\[4\]. The focus was a contentious political issue that had recently brought the mathematicians and politicians participating in the meetings into antagonistic (hostile) positions. Two short examples of the conflicts that arose:

Such postmodern experiences can also lead to discontentment and discomfort, as they often embrace dilemma when confronting contradiction. Here we may find, as a community of educators and researchers, that different members of our community...
disagreements over the legitimacy of the methods and findings of an economic impact study justifying the construction of a new bridge; and accusations that the underlying motivation for the bridge (and thus for the possibly faulty research study) was connected to a Jerry-rigged vote that (depending on one’s politics) supposedly or democratically led to the plan for a recreation of a former castle, which would require the relocation of an existing bridge. After media coverage of mathematicians confronting politicians at the construction site, politicians agreed on camera to meet with local youth to discuss the issues. This was surely a rare opportunity to research the formation of such curriculum experiences that would take place, as well as the cultural, historical and political context that made them possible.

understand our work in different ways. When it comes to representing and retelling about this work for each other and for people outside of our community, some point to conflicts. Deborah Britzman (1998) called this ‘difficult knowledge.’ Michael Apple (1979) once wrote of conflict as essential to democracy and learning. Dalene Swanson asks if we might find a possible peaceful/resigned discontent, or unnerving contentment?

Some traditional research questions were guiding this project: What knowledge is of most worth in such a project? The mathematics? The rhetorical uses of mathematics in social policy and political action? Tactics and strategies of political engagement? If we take seriously the notion that students and politicians bring a critical competence (Skovsmose, 2005) to educational encounters, then what features, components, aspects or characteristics of any potentially acknowledged or unacknowledged ‘knowledges’ could we or should we address in research with, about or around the curriculum events that emerge?

However, an overriding tension emerged, given that the effort was couched as a
mathematics curriculum project: On the one hand, any link between mathematics and
democratic participation is to be celebrated as making math relevant to everyday life,
introducing meaning and purpose into an otherwise isolated and rigid curriculum,
supporting interaction between youth and those who lead their government (which might be
thought to facilitate an increase in mathematics performance by the weight of this implicit
endorsement), and so on; On the other hand, any mathematical interpretation of the specific
contentious issues would likely support those mathematicians who had recently challenged
the intentions of the politicians and businesspersons who had recently pushed their agenda
through what might be described as a manipulation of a referendum vote, as well as a
biased and possibly unprofessional environmental and economic impact study.

In this research, attempts were made to avoid the fundamental modernist question: Who is
guilty? As well as its corollary: Of what are they guilty? Which, taken together, would
have implied an (unfortunate) internal logic unraveling through a narrative. For example, it
would hardly be rewarding to prepare students for meeting with the Oberbürgermeister of
Brandenburg (similar to the governor of a US State, or the premier of a Canadian Province)
by convincing them that the Oberbürgermeister had manipulated the vote, or that he and
others had Jerry-rigged the work and analysis of the environmental and economic impact of
their plans. Nor would it be productive to set up the mathematicians whose counter-
arguments were to be studied as guilty of performing political sabotage, or professional
suicide, or any other mix of motivation and ramifications of their work.

Moreover, it would not be useful to depict the business people as preternaturally evil; it
would not be productive or satisfying or coherent (or anything else we might associate with
a positive perspective on the research as ‘good research’) to combine our stories in such a
way as to imply that the mathematics educators leading the project were/are working for
one political party rather than another, or are/were brainwashing the youth about their
government or particular politicians, or were/are painting specific controversial public
decisions as grounded in a fundamental mathematical truth. Yet mathematics is typically
held up as the bastion of certainty and truth, making the layers of meaning and
interpretation unendingly recursive in the interplay between truth and meaning, cultural
context and political necessity. We only expect such storytelling in the curriculum because
it is associated with mathematics. The disciplinary label triggers expectations of truth and
certainty that are characteristically misplaced.

So what is this all about?
In my own version of Patti Lather’s (1996) troubling of clarity with and through refraction,
I began by juxtaposing the tales of the (educational) mathematical encounters with anti-
detective stories written in the context of mathematics (Beutelspacher, 2005; Drösser, 2007;
Fienberg, 2006; Lichtman, 2007; Martinez, 2008; Gatay & Bal, 2007). This is again why I
say that in a sense this publication represents a moment in my professional work. Various
strands of my professional life were folding upon one another seemingly by happenstance.
So, at the time that this anti-curriculum[5] project was proceeding, I was also continuing a
study of recent mathematical literature.

Have you noticed that the last few years have witnessed something of a renaissance in
fiction and nonfiction inspired by mathematical ideas? Most of these works are literature
for young readers (Appelbaum, 2008a), but many are also for adults. Many take the form of
mystery novels. I now own about ten books that proclaim to be the greatest popularization
of Infinity known to the published world, almost as many about Zero, plenty of well-
crafted tales of historical moments in intellectual history, novels, biographies,
autobiographies, historical fiction, philosophical entertainments, most of which have been the subject of newspaper and/or website reviews multiple times over, all focused on mathematics.

The novels, I was coming to realize, were very much in the postmodern literary style of a less than certain resolution. I had been working with the notion that reading such literature could be the primary, or only, materials of a curriculum, so that literature circles of learners would achieve traditional learning objectives by the time their circle completed a series of meetings and self-designed action projects growing out of the themes emerging in their discussions. The curricular point of such literature circles would have hardly anything to do with whether or not a book or two ended with a mystery solved and all tied-up in the end. But more to the point: the pleasures of reading have nothing to do with this ... An incomplete conclusion, an unresolved mystery, a mystery that turns out to be a non-mysterious triviality, or any other potential plot device, might add to the aesthetic, moral, political, or other value of the work, or not.

The analog is that the experience of an action curriculum project has little to do with learning or other categories of outcomes. However, the discussion—whether of detective fiction or curriculum research—is ‘found’ already located within a pragmatic epistemological frame: structural analyses are within this frame expected to demonstrate their conceptual worth in relation to contextualized aims and values. When we take away one or more of these presumed contextualized aims or values, we find that we still can preserve conceptual worth, unless of course our definition of worth is circularly defined with that particular aim or value in mind that we might have removed. In concert with Lather’s work with “troubling clarity” (1996) and her endorsement with Alan Block and others of “getting lost” (Lather 2007; Block 1995; Block 1998), we might seek something other than a structuralist conclusion, by interrogating the very centering of knowledge (for example, educational research, curriculum theory, pedagogical practice) via poststructuralist engagements concerning the limits of knowing and horizons of the unknown (i.e., Deleuze 1997, Derrida 1981, Doyle 1994, Serres, 1997).

However, the connection I made for myself, as I decided to proceed by juxtaposing narratives of the mathematics in democracy project with excerpts from contemporary postmodern mathematical fiction, was that there is a fine line between hope and accusation when one employs a narrative of mystery or explanation. I have always been heavily influenced by anthropologists such as George Marcus and James Clifford (Marcus & Fischer 1999; Marcus 1998; Clifford 2002; Clifford 2003), who began in the 1980s to make postcolonial ethnography central to the field of anthropology and other related human sciences, and in the process, walked a path of reflexive cultural critique as one result of intercultural research. Research is, in these narratives, as implicated in a Western ‘Will to Power’ as it is, sardonically, liberating, bifurcating, transcendent, creolizing, or any other potential ‘point’ or ‘purpose’ of that research.

Like the detective novel, the research narrative leads, in the sense of Derrida (1981), to an end that contains within itself the definition of that end’s opposite. What is the research leading to? This is a paramount question, which often feels like it is subsumed by the related question of how will the research get to where it is going, i.e., structure and method? As surreal art, research would have no need to obsess about any of this. But as academic work with a potential audience, it seems to call on that audience to be passive spectator, active critic, participant observer, or analytic experimenter (Ranciere 2009, Boal 1995). And what does this audience attend to? Structure? End? Purpose? Desire? Aesthetics? Catharsis? There appears, in other words, to be no point of research without a
desire for structure itself, whether our research strives to legitimate or enact structure or not—leading to confusion as to what a post-structure might entail.

I was drawn to several images: origami as the unfolding of folding upon itself; cat’s cradle (Haraway 1994); as constantly shifting constellations that require collaborative processes of give and take; and collage processes, where the imagined fractal contains within its self-similarity a recursive rule of relation that is more critical than the objects of study themselves. What each image foregrounds is the role of process in experience, with little attention to stasis or fixity.

Given the presence of mathematics in most of the narratives that are interwoven in this poststructuralist combination of folding and unfolding, knots and no-knots, we might claim that the work of a mathematician (that of making conjectures) was at the heart of much of the work in process. The structure of the conjecture rhizomatically evoked poststructuralist discourses in its determination to fold conclusion upon uncertainty, or we might say, uncertainty upon an alternative to resolution.

Stories
There was a time and a place when a civic leader was swayed by local sentiment to rebuild a palace of great renown. Since the grandeur of the land had dwindled, this palace had been demolished and replaced by schools, businesses, and housing for less prosperous members of society. Some people no longer saw schools, businesses and affordable apartments as a better symbol than a former relic of a long-ago royal residence, a home of monarchs who had led the land in honor. Those who had built the schools, businesses and apartment buildings had, since, been pushed out of power. They, in turn, had seized control from other leaders, who had done what world sentiment had called despicable. Now, citizens sought a new symbol of honor, and “the palace that once was” could serve this purpose.[6]

But many others thought that rebuilding a palace was folly. A vote of the parliament was called. The finding? No palace. Contractors and builders and crafts-workers and artisans were angry, and made it clear to the current leader that all was not right in his political base. What was he to do? Why, call for a referendum. But this referendum needed to be phrased in just the right way if the right result was to be realised. Options were carefully crafted, and the populace could not help but speak as wished: a new replica of the old palace would bring honor to the fair land.

Meanwhile, a young man was traveling with his aunt to a meeting of mathematicians in Italy, where local color mixed with international intrigue, and all awaited the earth-shattering discoveries of a famous mathematician. This mathematician had promised to reveal new secrets of cryptography, some of which might mean the end of global finance and consumer culture: any encoded message would be decipherable, even those currently assumed to be safe for internet transactions. But the professor giving this series of lectures was pitching his seminar meetings at the level of our young hero, rather than the sophisticated mathematicians who had assembled at this conference. And a socially inept upstart in the audience was apparently being set up as the accused in a murder attempt or equally felonious crime!

And the daughter of an artist is losing respect for her mother who seems too afraid to admit that a good friend of the family murdered his wife. All clues lead to this man’s guilt. Does this justify the obvious conclusion?
Students are set lose to arrange a meeting with the Oberbürgermeister, to interview citizens in their everyday life’s work. Local media smells a good story, a controversy. Video is set lose upon the land.

**Stop Making Sense**

Brent Davis (2008) recently wrote, “In the desire to pull learners along a smooth path of concept development, we’ve planed off the bumpy parts that were once the precise locations of meaning and elaboration.” We have, he says, “created obstacles in the effort to avoid them.” Davis describes “huh” moments, when it is possible to enter authentic mathematical conversations. For example, we might ask someone to describe what we mean when we write, “2/3 = 14/21.” Responses vary from pictures of objects to vectors on a number line, but all share a conceptual quality of relative change, so that increasing one thing leads to a proportional increase in another thing or group of things. However, when we ask the same person, “Describe what is happening in the expression, -1/1 = 1/-1?” We usually get a kind of “huh” response, which communicates a moment where the mathematics has lost its sense, but which also potentially begins an important (mathematical) conversation.

In my own work on what Davis calls the “huh” moments—when mathematics stops making sense to us, and we grope for models apparently not available (Appelbaum, 2008b)—I, too, have noted the potential for the non-sense-making characteristics of mathematics to generate different kinds of teacher-student relationships, and most significantly, different kinds of relations with mathematics within associated critical mathematical action (Appelbaum, 1999). Mathematics curriculum materials too often hide the messiness of mathematics where sense dissolves into paradox and perplexity; but what is more important is that they construct a false fantasy of coherence and consistency.

As most professional mathematicians understand, mathematics at its core is grounded in indefinable terms (Set? Point?), inconsistencies (Gödel’s proof? Cantor’s continuum hypothesis?), and incoherence (The limit paradox in calculus?). At a more basic level, multiplying fractions ends up making things smaller even though ‘multiplying’ conjures images of ‘increasing’ to many people; two cylinders made out of the same piece of paper (one rolled lengthwise, one widthwise) have the same surface area but hold different volumes; we’re taught to add multiple columns of numbers from right to left with regrouping in many cultures, when it is so much easier to think left to right starting with the bigger numbers. In some cases it is impossible, speaking epistemologically, for mathematics as a discipline to ‘make sense’—in others it might be more valuable pedagogically to treat mathematics ‘as if’ it does not make sense. To do so would celebrate the position of the learner, for whom much of the mathematics is new and possibly confusing anyway.

Yet, so much of contemporary mathematics education practice is devoted to helping students make sense of mathematics! What if we stopped trying to make sense totally, and instead worked together with students to study the ways in which mathematics both does and does not make sense? Rather than school experiences full of memorization and drill techniques, we would imagine classroom scenarios full of conversation about the implications of choosing one interpretation over another, or of explorations that compare and contrast models and metaphors for the wisdom they provide.

Elizabeth de Freitas (2008) describes our desire to make mathematics fit a false sense of certainty as ‘mathematical agency interfering with an abstract realm.’ She encourages teachers to intentionally ‘trouble’ the authority of the discipline, in order to belie the ‘reasonableness’ of mathematics. In this way our pupils and we can better understand how
mathematics is sometimes used in social contexts like policy documents and arguments, for business transactions, and philosophical debates, to obscure reason rather than to support it. Stephen Brown called this kind of pedagogy, “balance[ing] a commitment to truth as expressed within a body of knowledge or emerging knowledge, with an attitude of concern for how that knowledge sheds light in an idiosyncratic way on the emergence of a self” (Brown, 1973, 214). In other words, we can learn from mathematics instead of about mathematics.

So, you may wonder, what does this mean about curriculum materials and textbooks? “Obviously somebody somewhere with a lot of authority has actually sat down and written this Numeracy Strategy,” says one teacher with whom Tony Brown (2008) spoke, “it’s not like they don’t know what they are talking about.” Tony Brown blames the administrative performances that have shaped mathematics for masking what Brent Davis calls ‘the huh moments,’ and what de Freitas describes as the self-denial that accompanies “rule and rhythm.” Teaching in this “senseless world of mathematical practice” need not abandon science and the rational. It merely shifts teaching away from method and technique toward what Nathalie Sinclair (2006) calls the “craft” of the practitioner, as she evokes the metaphor of teaching as midwifery from Plato’s *Theaetetus* (see also Appelbaum 2000).

As midwives, teachers assist in the birth of knowledge; students experience not only the pain and unpredictability of the creative process, but also the responsibility for the life of this knowledge once it leaves ‘the womb.’ One must care for and nurture one’s knowledge, whether it acts rationally or not. Can we be confident that the ways we have raised our knowledge will prepare it for when it is let loose upon the world? Will our knowledge be embodied with its own self-awareness and ethical stance?

**A Dubious Theory**

A demand that everything make sense dominates the way we work with mathematics in school; this sense should be so simple that it virtually is instantaneously accomplished. We design a curriculum that introduces bits of new thought once per week, or even less often, because we worry that a learner will feel lost or confused and not be able to move on to the next tiny step that follows. I imagine instead a curriculum where students beg for new challenges, and where these students delight in the confusion that promises new worlds of thinking and acting, a situation with learners who do not just ‘get by’ in mathematics class, but who love mathematics as part of their sense of self and their engagement with their world.

The French philosopher and social theorist Michel de Certeau (1984) blamed the social sciences for reducing people to passive receivers of knowledge. Some might equally blame the natural sciences for the same thing. Indeed, educational research and practice has been dominated by ‘sciences’ for the past century, so we have been living the successes and failures of these approaches to education, and now we need to look at them critically as we reassess our work in mathematics.

De Certeau suggested that the social sciences cannot conceive of people as actors who invent new worlds and new forms of meaning because they study the traditions, language, symbols, art and articles of exchange that make up a culture but lack a formal means by which to examine the ways in which people re-appropriate them in everyday situations. This is a dangerous omission, he maintained, because it is through the activity of reuse that we would be able to understand the abundance of opportunities for ordinary people to subvert the rituals and representations that institutions seek to impose upon them. With no clear understanding of such activity, the social sciences are bound to create little more than
a picture of people who are non-artists (meaning non-creators and non-producers), passive and heavily subjected to ‘receiving’ culture. Social sciences thus typically understand people as passive receivers or “consumers” rather than as makers or inventors of culture, ideas, and social possibilities.

More recent social theory has wrestled with this in the context of structure versus agency, identity politics and the cultural politics of everyday life. Yet I am not sure how much this has informed educational research (See Willinsky 1999, 2000). Indeed, I believe this is exactly the situation we find ourselves in as we seek ways to make mathematics meaningful for young people and for young people to take advantage of mathematical skills and ideas (where necessary or appropriate) as they participate in their local and global communities.

This kind of misinterpretation is critical to our “consumer culture,” in which people are assigned to market niches and sold products, concepts, modes of life, and predictable desires. In curriculum, as in advertising, this social science paradigm persists to the point where we see students as consumers of knowledge whose desires are shaped by the curriculum via the teacher, where we see teachers as consumers of pedagogical training programs, and so on. De Certeau employs the word “user” for consumers; he expands the concept of “consumption” to encompass “procedures of consumption” and then builds on this notion to invent his idea of “tactics of consumption.” School curriculum tries to sell students on the value of mathematical knowledge; we sometimes call this ‘motivation.’ New curriculum materials are published and sold as part of a global economic system that demands new and improved products in a cycle of perpetual (built-in) obsolescence and ‘innovation.’

What would it mean for students who are learning “stuff that many teachers already know” to be artists—creators and producers—when we seem to want them to “consume and use” instead? Learners of mathematics experience the extra complication that much of mathematics is not even something that teachers and adults already know: part of the implicit character of the curriculum is that only some people know, as an example of natural gifts, or as a signifier of status. The critical notion turns out to be how we make sense of the “art.”

Susan Sontag (1966) wrote about what she named a “dubious theory,” i.e., that art contains content; a perspective that she claimed violates art itself. When we take art as containing content, we are led to assume that art represents and interprets stuff, and that these acts of representation and interpretation are the essence of art itself. Likewise, in school we often imagine the curriculum as content, and move quickly to the assumption that this curriculum represents and interprets. This makes art and curriculum into articles of use, for arrangement into a mental scheme of categories. What else could art or curriculum do? Well, Sontag suggests several things: “To avoid interpretation, art may become parody. Or it may become abstract. Or it may become (‘merely’) decorative. Or it may become non-art.” (Sontag 1966: 10)

**New Worlds of Mathematics Education**

Parody, abstraction, decoration, and/or non-art are four types of tactics for art and curriculum. I also think that they can be used to stop making sense of mathematics for young children, and to instead celebrate together those aspects of mathematics that do not make sense. I conclude this section of the article by exploring the application of de-Certeauian-Sontagian ‘tactics’ in four realms: Mathematics as a school subject; Teacher-training; Teachers’ work; and Learning Mathematics. With my suggestions, I am encouraging each of us to consider how school mathematics could be experienced as
something *other than* a representation of content, or something other than an abstract representation of ideas. This does not mean that I want us to abandon representations or the representation of ideas, but that our methods of teaching would not stress this as our primary purpose.

**Mathematics as a school subject:**

Normally, we emphasize two kinds of experiences in school mathematics, and through these we create an implicit story about what mathematics ‘is.’ We either 1) develop ideas out of concrete experiences, or we 2) model real-life events with mathematical language. An example of the first would be to work with numerals to represent numbers of objects, in order to stress for young children the differences between cardinality and ordinality [7] or to develop arithmetical algorithms for adding, subtracting, multiplying or dividing numbers. We might work with base-ten blocks, number lines, collections of objects, drawings of objects, and so on. An example of the second might be to create a story problem out of a real-life situation, such as to ask how many tables we need for a party if each table can seat six people and we expect fifteen people to attend our party; or to ask, given eighty meters of fencing material, what shape we should use to have the most area for our enclosed playground?

Now, suppose we wanted to transform our pedagogy so that the work in our classroom was one of parody, abstraction, decoration, or non-art, rather than representational art. Children might parody routine questions by acting out seemingly absurd situations where the reckoning leads to ludicrous results, or they might ask and answer questions that shed humorous or critical light on typical uses of the mathematics. For example, 4-year-olds who have counted the number of steps from their classroom to the door of the building in ones, threes and fives, might then count the number of drops of water to fill a bucket in ones, threes and fives, even though it seems to make no sense to do so … this would only be a parody, though, if the children themselves suggested it as a silly thing that they wished to do nevertheless. Similarly, ten-year-olds might design alternative arrangements of their classroom that make use of unusually shaped desks, such as asymmetric trapezoids, circles, etc.

Mathematics might be abstract, in Sontag’s sense of avoiding representation, if children did more comparing and contrasting of questions, methods, and types of mathematical situations, rather than focusing on the particular questions or on practicing specific methods. For example, 8-year-olds or eleventh grade students might organize collections of mathematics problems first into three categories, and then the same problems into four new categories, rather than solving the problems themselves; the classification of the problems into types would constitute the mathematical work, rather than the solution of the mathematical problems.

Mathematics as ‘decoration’ might be accomplished through a classroom project where students experiment with different representations of a mathematical idea for communicating with various audiences. After working with ratio and proportion for example, a class of 11-year-olds might form small groups, one of which creates a puppet...
show for younger children, one of which composes a book of poems for older children, and another of which prepares a presentation for adults at their neighborhood senior citizens community center, all on the same subject of applying ratio and proportion to understanding the ways that a recent election unfolded. In this sense of considering the appropriate way to describe ratio and proportion for a particular audience, the mathematics is more of a decorative form of rhetoric than a collection of skills or concepts; the important concepts have more to do with democratic participation in elections than with the mathematics per se.

Mathematics as non-art uses artistic work that is not considered ‘art’ as its model—we could ask, when is creative mathematical work not mathematics? One answer is, when it is something else other than mathematics per se—for example, when it is an argument for social action presented at a meeting, or when it is an example used to demonstrate a philosophical point, or when it is a recreational past-time, etc. In other words, mathematics as non-art would be mathematics that isn’t done for its own sake; mathematics as non-art would be mathematics for the purposes of philosophy, anthropology, literature, poetry, archaeology, history, science, religion, and so on—as long as the activity has a purpose other than the mathematics itself.

Teacher’s Work:

So, mathematics as a school subject can and should take on the character of parody, abstraction, decoration or non-art. But if this is to occur, there are important implications for the teacher’s work. For one thing, the teacher would not be providing clear presentations or explanations of mathematical concepts or procedural skills. With regard to this prospect, we can learn from current work at the University of Amsterdam on the types of teacher help that support mathematical level raising (Dekker & Elshout-Mohr 2004, 2005; Pijls 2007; Pijls, Dekker & Van Hout-Wolters 2007). In their studies, they have found that teacher help directed at mathematical content—explanations and demonstrations—is rarely more valuable than teacher help directed at collaborative learning and groups processes, and in fact, sometimes teacher help that is focused only on the group processes leads to more significant conceptual level-raising.

In other words, the nature of useful teacher work involves making it possible for pupils to participate as creators and consumers of nonrepresentational mathematical art, which does not aim at simplifying the path to sense making. Instead, teacher work essentially makes it possible for learners to experience together the authentic practices of sense and non-sense through events such as parody, abstraction, decoration, and non-art. In the group processes that are supported by teacher-help, the mathematics is secondary to the group process in the teacher’s mind. The teacher is helping the learners to use mathematics in order to accomplish the group process, rather than using organization of the group in order to accomplish representation or sense-making of mathematics. This seems backward, given that our job is to teach mathematics! It is almost counterintuitive! But indeed, when we think this way, perhaps, there is a new ‘sense’ to be made of mathematics teaching.

Teacher training:

What then, are the implications for teacher training? I believe the key things to think about are the differences between preparation for representation and sense-making, which has been the primary direction of mathematics education for the last century, and preparation for the support of artistic practice. We have inherited a technology of teaching methods steeped in cognitive psychology, which direct the teacher’s attention to individual cognitive development. This has certainly been useful, and will continue to be useful to all of us in our work. However, my experiences with the Mathematics in Democracy project helped me
to see the differences between an artistic use of mathematics to illustrate an argument, and the uses of mathematics to perform irony or parody.

Students in Potsdam tried to make sense of an official economic and environmental impact study. The report used mathematics to convey a logical closure to the wise plan to build a new bridge next to an older one. In embracing the ways that the report’s use of mathematics did and did not make sense mathematically, students were able to form opinions about the report as a performance of irony when they noted ways that nonprofessionals (of mathematics) could easily produce a more effective argument with the same conclusions, or that professionals could easily make a parallel argument with opposite conclusions. The students were also able to experience the report and their own discussion of it as parody when they placed the mathematics as a tool and as rhetoric in multiple social contexts. An aesthetic valuation of mathematics might value a work of mathematics as art if we place art that communicates ideas through a performance (embodiment) of those ideas above art that communicates ideas by illustrating (i.e., representing) them.

I foreground another orientation to this work, which Eliot Eisner (1991) called criticism and connoisseurship. Ordinarily, the teacher training that I am most familiar with involves extensive practice in the application of methods, diagnosis and remediation. Eisner’s ideas suggest instead that teachers-in-training spend more time immersed in experiences that are not directly focused on the representation of teaching and learning, or on making sense of what pupils can and cannot do, but instead on criticism and connoisseurship in the context of schools.

Connoisseurship is the art of appreciation. It can be displayed in any realm in which the character, import or value of objects, situations, and performances are distributed and variable, including educational practice. The word connoisseurship comes from the Latin *cognoscere*, to know. It involves the ability to see, not merely look. To do this we have to develop the ability to name and appreciate the different dimensions of situations and experiences, and the way they relate one to another. We have to be able to draw upon, and make use of, a wide array of information. We also have to be able to place our experiences and understandings in a wider context, and connect them with our values and commitments. Connoisseurship is something that needs to be worked at—but it is not a technical exercise. The bringing together of the different elements into a whole involves artistry.

It may sound like I am advocating an elitist notion here, but I do not mean this; indeed, I want us to think mainly about the depth of knowledge that all people have in their everyday lives as connoisseurs of those things they taste deeply, and to imagine how we could help young people to take those ways of learning and thinking and making meaning, and see that they are relevant in school (Gustavson & Appelbaum 2005; Appelbaum 2008a). Now, what Eisner makes clear in his writing is that educators need to be more than connoisseurs. They need to become critics. Our models for ourselves need to be reviewers of the films, albums, music videos, and video-games that we read and listen to for pleasure, and that help us to know which artistic works we will enjoy and find valuable, including even those critics with whom we love to disagree. Criticism is the art of disclosure, of revealing more than the obvious. As John Dewey pointed out in his book *Art as Experience*, criticism has as its aim the reeducation of perception.

Thus … connoisseurship provides criticism with its subject matter. Connoisseurship is private, but criticism is public. Connoisseurs simply need to appreciate what they encounter.
Critics, however, must render these qualities vivid by the artful use of critical disclosure.

—Eisner 1985: 92-93

In my own work in teacher education, I strive to work as a connoisseur and critic, in order to support the artistry of my students who wish to be teachers. And I welcome conversations with you over coffee, tea, a beer, wine, and so on, to share such stories. But back to the main theme of this section: what sort of mathematics learning is enabled by a teacher with extensive background in connoisseurship and criticism? Experiences taking the role of instigator of parody, playing with abstraction, critiquing decoration. Reflection on work as a facilitator of mathematics as non-mathematics, as in the Mathematics for Democracy project during their country’s Year of Mathematics. Teacher training is apprenticeship[8] in connoisseurship.

**Learning Mathematics:**
Well, we could simply say that learners of mathematics are succeeding when they are demonstrating abilities to use mathematics in order to achieve a parody, to communicate an abstraction, as a decorative element in other contexts, or as non-mathematics across the curriculum. But more directly, I offer the following: Young people learning mathematics are artists whose tactics of parody, abstraction, decoration and non-art are forms of consumption that re-appropriate school mathematics as a tool of connoisseurship and thus, of remaking their world anew in each act of mathematics they commit. Here is a very active and vibrant way to imagine mathematics learning: as artistry, as doing, as alive, and as transforming the world in every tiny moment. Mathematics in this ‘sense’ is a collection of tactics for doing this. And learning mathematics is an apprenticeship in the artistry of social participation.

Students’ and teachers’ mathematical actions, as art, are not aimed at a purpose that involves curricular illustration, but instead become the embodiment of a critical pedagogy that engages both the mathematical artist and the artistic mathematician in critical citizenship (Springgay and Freedman 2007). The challenge, then, becomes: are we ready to allow the children in our life and work to become connoisseurs of mathematics? That is, to become more than ‘knowers,’ to become critics of mathematics? Mathematics as criticism is an art of disclosure, of revealing more than what is obvious on the surface.

Here is the magic recipe for achieving this: think more about coordinating activities where the children are active artists of mathematics than about how to represent or explain clearly a mathematical concept. I know it goes against so much of our desires to make things easier for the child. In the end though, if we stick to this plan, we will be lucky enough to spend time with current and future crafters of beautiful worlds, young people who use mathematics to shed insight on contemporary society, to ironically critique common sense practices, as a tool for appreciating and interpreting culture and societal problems, as the medium of decoration and entertainment, and simply as something so valuable as to be part of all things not usually named ‘mathematics’.

**Ironic Ending**
Clues lead to guilt? Well, clues are not evidence. If Mom suspects possible foul play, why doesn’t she share this with the police? How could she withhold this? Curious indeed, given the young woman’s propensity to interpreting almost everything in her life according to mathematics. But we see that mathematics leads to emotion and desire, whereas the artist mom more clearly makes the distinction between clues and evidence. Undetective readers
of *Christian und die Zahlenkünstler* see the clues all along. The professor is stalling. He has no world-shattering mathematics to offer. The real crime will have to be a faked break-in with the result that the ‘manuscript’ is gone. But the undetective knows this is not the real crime. If crime is associated with guilt, the guilt lies at the heart of academic life, with its constant pressure to produce completely new, never-before-seen, awe-inspiring mathematics. The professor is as much a victim here as the seemingly guilty upstart who suspected the crime all along.

In the folly of the rebuilt palace, one might eagerly assign guilt to the civic leaders, desperately overspending public funds in the service of honor. But the real crime, if there is one to be found, lies in our political systems, which demand of our politicians that they become identified with a politics of aesthetics (Appelbaum 2008c), which functions as much on the surface of imagery and symbol as it does on the integrity and reliability of environmental and economic impact studies.

In the Mathematics in Democracy project, students quickly noted the weaknesses in the report, and wondered how anyone could be going ahead with the extremely expensive, associated plans. Surely those hired to perform this study would be expected to maintain standards of professional ethics that would not allow this folly? How had the Oberbürgermeister and other politicians been able to dismiss the alternative analysis of data submitted by the mathematics professor, a report that presented contrary conclusions? The Oberbürgermeister was captured on video declaring “you have your own opinion, and I have mine,” as if the mathematics were a mere matter of personal preference. If this is the case, then why pour so many public funds into the extensive impact studies in the first place? If one is to use such a study to justify a large-scale public works project, then surely one lays oneself bare to the critique of that study and its authority. Given our societal cult of expertise, shouldn’t the mathematician’s analysis of the data hold even greater weight than that of the research firm hired by the politicians, possibly under much pressure to produce a report favorable to the politicians who voted to fund the firm? And in a ‘Year of Mathematics’ can we not seize this opportunity to explicitly discuss the role of mathematics in our democratic process?

The secondary school students proposed several readings of the report: As a parody of good government, in which the best government is one that best enables itself and its friends to benefit from its hold on power; As a reading of mathematical decoration, in which the mathematical analysis serves to provide a rhetorical flourish to a foregone conclusion; As an abstraction, in which the mathematics was taken into new territory of alternative variables, modifying conclusions under varying constraints and weights of measure; And finally, as a non-mathematical story of ignorant social scientists needing to learn more school-level mathematics. (After all, much of the data involved finding methods to count the number of cars estimated to be traveling through the streets at different points on a map, and other mathematics applied standard weights of value to various social costs created by new traffic patterns. These are often considered elementary or primary school topics.)

What these students did not address, but which is important for curriculum studies in general, is the ways in which mathematical narratives communicate a certain kind of arrogance in helping to constitute ‘others’ as ‘outside the domain’ through notions of ignorance. The curriculum can, in this sense, foster an interpretation that carries with it a socially constructed binary of a ‘knowing’ versus an ‘ignorant’ subject. These very problematic implications were, to come full circle, at the initial core of our research concerns about the passive mathematical audience versus the active democratic citizen-mathematician.
But our point is not to create a curriculum (or an article such as this one) that is a ‘machinery of interpretation.’ We cannot represent the curricular experience, or its political context, as a form of mathematics for/in/with democracy. We should not illustrate the point of it all; that would be to lay blame on some specific aspect of our representation or illustration as guilty of a specific catalytic causation. Such blame is nothing more than a “fantasy of transcendental origin, an ultimate guarantor of Truth unsituated in time, space, or history.” (Rotman 1993: 157) Which brings us back to the notion that even when we conjure up an unbounded infinity of infinities, they play out as if they were little more than isomorphic to our basic counting numbers themselves. If research is an emptiness to be filled with wisdom, if mathematics curriculum is limitless possibility, then these things are fantasies of transcendence. And if we insist on playing within these fantasies, then what we accomplish are reconstructions of our human perception of time and space as a continuum with ordered dimensions: we can conceive of ever-increasing dimensions, but the dimensions of these infinite possibilities remain at the same cardinality. The fantasy is that we can create a model of our experience that would allow or enable infinite, reflective thought. Abstract art, on the other hand, offers another way of thinking, by freeing us from the stranglehold of representation. The sculptor Josiah McElheny suggests we might begin to understand this by comparing the ways that artists and architects use models. In his work, the artist is not encumbered with the physical, economic, political and other constraints that architects celebrate as making their work possible (McElheny 2007).

Perhaps another useful metaphor is to compare the research project or the particular curriculum, as historian John Murrin (1987) once described the US Constitution, as a “roof without walls.” Murrin writes that the Americans had erected their constitutional roof before they constructed national walls. Hovering there over a divided people, the roof aroused wonder and awe, even ecstasy. Early historians rewrote the past to make the Constitution the culminating event of their story. Some of the early Republic’s most talented legal minds composed long-winded, multivolume commentaries on its unbounded virtues and limitless wisdom. Murrin writes, “Orators plundered the language in search of fitting praise. Someone may even have put the document to music. This spirit of amazement, this frenzy of self-congratulation, owed its intensity to the terrible fear that the roof could come crashing down at almost any time. Indeed, the national walls have taken much longer to build.” And so it is with any fantasy of infinite reflection.

Contrast a Jewish Sukkah, a temporary shelter-like construction built for the weeklong holiday of Sukkot. Intentionally impermanent, one must theoretically use two complete walls and part of a third for a structure to qualify as a Sukkah according to religious law, although it is common to have four walls. Far from a roof without walls, the Sukkah has a covering through which one can see the stars, composed of material that grows from the soil. The Sukkah should not be more than 20 cubits (about 30 feet) high, since then it could become a temporary dwelling; it should be at least ten handbreadths (three feet) high and provide enough space to accommodate at least one person. A model of a dwelling that is not a dwelling, the Sukkah is perhaps the earliest known example of what McElheny is trying to describe: a builder of a Sukkah creates a model of a place-to-be that is not an architect’s model of a place to live. Because of this, it generates possibility. It is neither an illustration of a dwelling, nor a representation of a home. Its design does not determine what happens within or without. It is instead a space that provokes questions about its existence, due to its function as a parody of temporary shelters traditionally used during harvest seasons, its service in holiday traditions as a decorative feature of cultural and religious observance, its abstraction as a conceptual piece of cultural,
religious, meaning, and its ambivalent role as a work of aesthetic appeal with no apparent utility, in this latter role as a non-structure. Lacking complete walls and a secure roof, the Sukkah denies the possibility of an enclosed space of infinite experience, instead it is performing the blurred boundaries between inside and outside, human artifice and nature, shelter and fragility. Rather than representing a concept, the Sukkah performs its meaning.

And so should research and curriculum be non-illustrative, non-representative performances of meaning that blur boundaries of inside and outside, human artifice and nature, shelter and fragility. Research and Mathematics curriculum as abstract art—as parody, abstraction, decoration, or non-art—can make democratic forms of participation possible by performing rather than illustrating meaning. This turns out to be the way I experienced the Mathematics in Democracy project in Potsdam. As a curriculum, it shared Sontag’s characteristics of nonrepresentational art. The project was a parody of school mathematics. It took place during four meetings: one visit to a secondary school classroom to invite students to take on this project as a way of fulfilling their graduation requirements, and three full school days—none of which looked or felt like a typical school mathematics lesson.

The project was abstract: all efforts were made to suspend interpretation in favor of the direct experience of particularities. Students studied the economic and environmental impact report, the alternative analysis performed by the mathematician, and the subsequent actions taken by their State government. They scoured the streets for people they could speak with about the new palace and associated bridge, once the news of the groundbreaking of the construction site was well known, and pursued a concrete appointment with the Oberbürgermeister, as per his public promise to meet with local youth. The project was decoration in that it made the students look good to be carrying out such a study connecting mathematics with democratic practices during the country’s ‘Year of Mathematics.’ As an add-on to the regular curriculum, it served more to make the school look like a ‘cool’ place where such interesting things happen than as a mathematics curriculum unit. And it was non-school in that nothing ‘counted’ for their successful completion of their graduation, other than simple participation for the duration of the project.

As mathematics, the Mathematics in Democracy project also shared Sontag’s characteristics of nonrepresentational art. The mathematical content directly performed questions about the nature and role of mathematical models in creating meaning out of social processes, rather than illustrating mathematical truths by representing ‘reality’ in such models. Students experienced mathematics embedded in the political process, instead of learning it as content to digest, because of the arbitrary selection of materials, idiosyncratic mathematical connections, and complete lack of intentional interpretation in the facilitation of the experience.

Indeed, there was no possibility to directly help the youth ‘make sense’ of the mathematics in the report, because the fundamental mathematical issues seemed to have more to do with the creation of the mathematical model and its subsequent use than any particular mathematical techniques or concepts. The mathematics of the report was at a low, elementary level, despite the advanced nature of the mathematical modeling processes involved. The mathematics was in fact decoration, since the ‘real issues’ were not the mathematics per se, but rather the uses of mathematics by professionals, the interpretation of the mathematics via official documents by politicians, and the subsequent concern to be prepared to make the most of a rarely obtained meeting with the Oberbürgermeister. Finally, the mathematics was non-mathematics, given that the project days focused on the
politics of mathematical models rather than school mathematical content.

Such nonrepresentational artistic practice is more easily experienced by the researcher as an undetective than by the crime-solving detective researcher. However much we hunt for the clues to whom or what is ‘guilty’ of creating a successful educational experience, we are left in the end with no resolution. Yet the rollicking ride made for a good story despite the lack of conclusion. “Writing,” writes Noel Gough, “is a method of discovery, a way of finding out about yourself and our world.” (Gough 2007: 208) This article was itself an experiment in writing as method, with the extra intent of avoiding representational writing. Gough claims that writing as a method of inquiry honors and encourages the trying, recognizing it as emblematic of the significance of language.

Whereas the detective works to unravel complexity in the service of order and representation, the undetective uses detection more as temporary markers of the processes of reconfiguration. Undetectives are led to new beginnings instead of closure. Their writing is like snapshots of cat’s cradle aficionados’ strategic thoughts. I took Gough seriously, and worked here to use writing in that way, as an artistic practice. Can you see that I was working to perform the meaning rather than illustrate it? I felt like a card-carrying member of the ‘International Educational Undetectives Association.’ If you would like to become a member as well, please contact me at Appelbaum (at) arcadia.edu.

---

Notes

[1] For example, if our set looks like this: \{1,3,∅, A\}, then the following sets are subsets of this set: \{1\}, \{3\}, \{∅\}, \{A\}, \{1,3\}, \{1,∅\}, \{1,A\}, \{3,∅\}, \{3,A\}, \{∅,A\}, \{1,3,∅\}, \{1,3,A\}, \{1,∅,A\}, \{3,∅, A\}. We could also say the set itself is a subset of the original set, trivial or redundant though it may be, if we say further that a set is a subset when everything in it is also in the original set. Mathematicians also include the empty set, \∅, since nothing in the set of nothing is not in the original set—which may be silly, but it is a convention that works out well in other situations. At any rate, we can see that a set of 4 things ends up leading to a power set of 2^4, or 16 things.

The power set is the set of subsets: \{\{1\}, \{3\}, \{∅\}, \{A\}, \{1,3\}, \{1,∅\}, \{1,A\}, \{3,∅\}, \{3,A\}, \{∅,A\}, \{1,3,∅\}, \{1,3,A\}, \{1,∅,A\}, \{3,∅, A\}, ∅, \{1,3,∅, A\}\}. It makes sense, then, that the set which has all of the possible combinations of the elements of another, original set, would be bigger, with more elements in it.

The explanation for sets in general, and not for just this case of 4 items in the original set, also applies to really big, and infinite sets, for which we could never be able to write down all of the combinations.

[2] Dalene Swanson recently noted that some researchers tend to engage in research by starting out assuming that they are (morally and rationally) right; they then go and prove that they are right, because they are right. This is indeed another story form of research, often derisively labeled “circular reasoning.” [http://www.toothpastefordinner.com/012408/circular-reasoning.gif]
Here is a mathematical example of what Cantor was thinking about: We might consider the counting numbers, \{1,2,3,4,5,\ldots\} and compare them with the set of fractions, and the set of all numbers on the number line. It seems like there are many more fractions than counting numbers, but Cantor found a clever way to list them so they really could be thought of as ‘listable,’ or countable. By matching the first one in the list with the number 1, the second with the number 2, and so on, Cantor showed that the infinite size of the counting numbers and the infinite size of all fractions can be considered the same—the sets have the same cardinality.

That clever list was ingenious. But even more amazing, he used that \textit{reductio ad absurdum} method to show that we can never list all the numbers. For an explanation using decimals, see the Wikipedia entry [http://en.wikipedia.org/wiki/Cantor’s_diagonal_argument] on this topic. The question comes up: if countable sets and uncountable ones like all the numbers on the number line (sometimes called ‘the continuum’) are different kinds of infinity, are there more kinds of infinity? Might there be infinities between the two we have found? Cantor’s intuition was that there are no infinities between the two we are discussing here; he wondered if the continuum might simply be the power set of the countable counting numbers.

Many thanks to Dr. Wolfram Meyerhöfer, then of the Freie Universität Berlin, currently Professor of Mathematics Education at the Universität Paderborn, for making the project possible, for taking the primary role of facilitating the meetings with the students in Potsdam, and for acquiring funding to support my own participation in the project.

Aoki (1999) mentions “the tensioned textured spaces” between curriculum and not curriculum, between planned and not planned—spaces between/outside/within; “such a human site promises generative possibilities and hope. It is, indeed, a site of becoming, where newness can come into being. The space moves and is alive!” (181).

The specific story is post-unification Germany. One interpretation of recent events is in terms of an attempt to obliterate both the Nazi and communist periods by covering them up with a romanticized notion of pre-WWI Kaiserreich monarchism. Initial votes to rebuild the palace did not go ‘the right way’—mathematicians claimed that the subsequent referendum was ‘rigged’ to support the palace reconstruction, because it split the ‘no’ votes across several options, whereas the ‘yes’ votes were consolidated into one option.

Cardinality refers to the need to understand that numbers name something about quantities; ordinality refers to the specific \textit{order} of numbers, including relative size.

Here I am using ‘apprenticeship’ in the sense of Ladson Billings (1995), who compares apprenticeship of students who bring funds of knowledge to the educational encounter, with
instruction of students who are treated as if they have no platform for learning new material. This should not be confused with conceptions of apprenticeship that imply a ‘knowing subject’ apprenticing an ‘unknowing subject.’

References


