Teaching and Learning Mathematics Through Discourse

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Opening thoughts
This paper is an extension of a piece that I recently presented as part of a collective project at the Provoking Curriculum Conference (Miranda, 2003). The collective project mainly grappled with a question of what counts as knowing, learning or teaching. This prompted me to question what counts as knowing in discourses that arise in mathematics classrooms. Here, I explore the aspect of mathematics classroom discourses by looking at the question of what one may pay attention to when observing such discourses. First, I present a summary of evidence on mathematics educators’ and researchers’ interests in teaching and learning mathematics through discourse. I also discuss the motive behind the researchers’ interests—i.e. why we should encourage discourses in mathematics classrooms. Second, I look at mathematics students’ co-participation in classroom discourses as one of the ways in which they may be actively engaged in generating mathematical knowledge. Third and lastly, I present a view of learning as a collective activity. I look at how this view can help in understanding the discourses that spontaneously emerge in the mathematics classroom and in understanding the autopoietic nature of a classroom.

Current trends in mathematics educational research
Over the past few decades, educational systems worldwide have been and still are going through several reforms. Such reforms include the prominent move from the so-called teacher-centered education to learner-centered education. In North America, for example, the recent reform in mathematics education, as part of this universal move, is the one that is inspired by the constructivist theories on learning. Moreover, in many cases, learner-centered education has been treated as synonymous to
I am, therefore, mentioning learner-centered education and constructivism here because both these two schools of thought emphasize students’ active engagement in/with the teaching and learning process. This paper is about students’ co-participation in generating mathematics discourses, which is one of the ways to actively involve students in the teaching and learning process. In particular, a learner-centered education requires a “communicative and language-rich mathematics classroom” (Adler, 1998, p. 24) as compared to the traditional approaches in which most talk is done by the teacher alone. Therefore one needs to consider the discursive conditions in which these requirements can be met in any mathematics classroom.

Constructivists stress that students should be allowed to actively engage in the construction of knowledge and their own understanding (Richards, 1991; Confrey, 1991). From this perspective it is asserted that students learn best when engaged in learning activities that allow them to communicate their thoughts and understanding and try to understand their peer’s and teacher’s points of view. It is thus believed that learning mathematics through discussions and conversations is important and plays a role in students’ mathematical development (Cobb, et al., 1998). However, as Sfard (1998) argues, a mere belief that learning through conversation is important is not sufficient in making meaning of how students learn mathematics. The question to be posed should therefore be “not whether to teach through conversation, but rather how” (p. 50, italics in origin). It is thus through proper observations of how such learning takes place that one could inform his or her theories on how to teach mathematics through discourse. Lesh, Lovitts and Kelly (2000) also write: One of the most important current needs in basic research on student learning processes is the need for insight explanatory models of these processes (p. 23).

Engaging students in mathematical discourses is believed to be one way of enhancing social learning in the mathematics classroom. Therefore paying attention to the emerging discourses and any other social behaviors may contribute to students’ development of mathematical argumentation and reasoning (Yackel, 2000). Several researchers and authors have discussed how small group and whole-classroom discussions can improve the learning and teaching of mathematics (Davis 1990; Yackel, Cobb, Wood, Wheatly, & Merkel, 1990).

Involving students in discourses also challenges the notion of the teacher’s role as the sole owner of authority, information provider and answer verifier in the mathematics classroom (Richards, 1991; Cobb, 1999; Yackel, 2000). It “challenges the image of a silent mathematics classroom where the talking by students has been limited to one word or numerical response to teacher-directed questions” (Gordon Calvert, 2001, p. 45). To a
certain extent, authority is and should be distributed and shared by all participants across the classroom. However, as Davis & Simmt (in press) advise, this distribution and sharing of authority should not be taken as a matter of “anything goes.” Rather it is worth understanding that, “with the emergence of any complex collective, standards of acceptable activity—of rightness and wrongness—inevitably arise” (p. 10). Having said that, it is also worth noticing that authority in the mathematics classroom does not only reside within the actions of deciding what and who is right or wrong but also with the notion of who or what creates the knowledge and ideas that become part of the collective. Therefore allowing students to talk about mathematics and about their talks about mathematics (Yackel, 2000) can be one of many ways in which students are invited into the community of school mathematicians and hence experience the sense of mathematical authorship.

**Students as co-participating mathematicians**

Following from the above paragraphs is the view, held by the situated cognition theorists, of learning as a “social co-participation” in a community of practices that is directed toward expertise and determined by the cultural and social context in which it is enmeshed (Lave & Wenger, 1991). They call this process of co-participation “apprenticeship” under the supervision of an expert. In a classroom discourse, for example, a teacher or other (but advanced) student may act as the expert to an individual student or a group of students who do not yet know much about a topic under discussion. From this social, mutual interaction both the apprentice and the expert learn. This co-participation also plays a role in students’ understanding (Confrey, 1995), for their cognitive processes are more effective in the presence of others (teacher or other students), especially “the others whose competences are more developed” (Davis, et al. 2000, p. 67) than their own.

By saying this, however, I do not view students as apprentices pretending or expected to pretend to be mathematicians (Davis & Sumara, in press). For when we are teaching we are not preparing students to become mathematicians by allowing them to mimic what the mathematicians do but rather occasioning them to be mathematicians in their current mathematical context (Gordon Calvert, 2001). Therefore, instead of looking at learning as a preparation for future practices in the field of mathematics, learners may be allowed to practice school mathematics by engaging in mathematical discourses.

The concern among mathematics educators about the role of language in mathematics learning and teaching has a long history (Wales, 1984). However, current researchers express that there is still a “need to redefine the role of language and its significance for mathematics lessons” (Abele, 1998). Mathematics education researchers and educators have recently indicated interest in the nature of mathematical discussions and
conversations (Davis, 1990; Gordon-Calvert, 2001; Davis & Simmt, in press). In such studies and literature, mathematics classrooms are observed and presented as communities of novice mathematicians talking about mathematics. Engaging students in bringing forth mathematical discourses—either in small groups or as a whole-class—is one of many ways in which students can be involved in the knowledge generating process. It has been noticed that through discussions, students share, clarify and consolidate their mathematical ideas and thoughts (Davis, 1990). In addition, it is through having meaningful discussions that students can be introduced to the community of mathematicians under the supervision of the teacher (Lave & Wenger, 1991).

**Collective learning through discourse**

Traditionally, mathematics learning is viewed as an individual process that occurs in an individual head and is affected by external factors such as the environment. A new challenge by current trends in mathematics educational research is the one that views learning as a collective activity (Kieren, 2000; Davis & Simmt, in press). From this point of view, teaching and learning is not understood as the responsibility of the teacher alone or understood to be in the head of an individual student but rather as the responsibility of the collective in which it co-emerges (Kieren and Simmt, 2002). The individual is not viewed as the sole unity of cognition but the whole group as a collective and the mathematics that emerges from the interactions within the collective. It thus follows that mathematics teachers, educators and researchers could pay close attention to the complex classroom behaviors that are potential for emerging discourses to support collective learning. One could attend to how the discourses emerge as the ideas brought forth "bump into one another" in order to understand collective learning in a mathematics classroom (Davis and Simmt, in press).

**Deliberate and spontaneous discourses**

In most traditional mathematics classrooms, discourses that are counted are usually the ones that are deliberately prepared and directed by the teacher. What is problematic in these practices is paying exclusive attention to the discussions directed by the teacher while discussions that spontaneously emerge are ignored. Such spontaneous discourses are often treated as off-task talk, off-topic behaviors, etc. My thesis here is that no discourse is off-task or off-topic. Since it arises within a community while engaged in a particular discursive practice, it has to be related to that practice in one way or the other. An emerging idea may appear off-task to an observer depending on how such an observer treats mathematics. Gordon Calvert (2001) discusses two ways in which mathematics can be thought of: mathematics as a finite game and mathematics as an infinite game. To those who view mathematics learning as a finite game, spontaneously emerging discourses appear as "a diversion or as off-task or off-track behaviour because it is at odds with the purpose for engaging in
mathematics” (126). Mathematics could therefore be viewed as a finite game in which spontaneously emerging discourses may serve to “[stretch] boundaries to include new actions, new interests, and new concerns” (128).

A mathematics classroom as an autopoietic system

The discussion of spontaneously emerging classroom discourses could be understood in Maturana and Varela’s notion of autopoiesis. In the Santiago Theory of learning [5], Maturana and Varela view cognition—i.e. the way of knowing—as a process of life. Cognition, according to them, involves such processes as self-organizing, self-updating and self-perpetuating which all have their roots in Maturana’s notion of living organisms as autopoietic systems. Autopoietic systems learn by making distinctions and learning mathematics is about making distinctions. Mathematics classrooms as well as groups and the mathematics that arise within them can be viewed as complex, self-organizing and self-updating, autopoietic systems (Davis & Simmt, in press). Even though the teacher comes to the classroom with deliberately prepared issues or topics for discussion, the emerging discourses might take a different direction to what she or he anticipated. This is an example of the autopoietic nature of the learning collective as a living unit on its own, for which the teacher like his or her students is just an “organ,” albeit a more crucial organ. Paying attention to not only the deliberate interactions among learning individuals but also to the spontaneous, structural behavior of the social learning unit may therefore allow room for the observation of ignored or unrecognized/taken-for-granted learning behaviors in a mathematics classroom (Namukasa, in press).

Understanding the classroom as an autopoietic being might give the researcher as well as the teacher and the students a chance to pay attention to the emerging properties of the collective to which they are just organs that can in many ways increase the emergence of artefacts, ideas and projects that transcend possibilities of them as individuals. The notion of learning as doing also allows them as teachers and students to adopt the stance that cognizing systems (including the individuals, sub-collectives and co-collectives) learn by doing the mathematics. Discussions, therefore, may not mean just talking about the subject but rather doing or making it. Learning systems only learn to ensure their fit with a co-evolving world through structural coupling.

In Maturana’s words: “Human beings talk about things because they generate the things they talk about” (1978; p. 56). This quote can be interpreted in many possible ways of which I will outline two. First, from a constructivist point of view, human beings are able to talk about ideas that they generate or construct and that consequently grow into potentially meaningful discourses. It is this ability that allows the collective to keep the discussions going and to generate more and new ideas that may condition future related discussions and conversations. For instance, in
mathematics classroom discourses, students ought to discuss ideas, thoughts, and doubts that they generate but not only to talk about ideas that someone else—i.e. a mathematician—has developed for them. Second, a more radical interpretation of the same quote, above, can be interpreted from views of those who look at classroom discussions and the mathematics itself as evolving phenomena (Cobb, Stephen, McClain & Gravemeijer, 2001; Gordon-Calvert, 2001) of the learning system. When one talks of students interacting with one another or with the teacher, one not only means interactions between students as talking bodies, but also the interactions between the generated ideas as they bump into one another (Davis & Simmt, in press) and taken up for discussions. Each emergent idea, therefore, has a potential to generate or evolve into other ideas, which may also unfold into further discussions.

Concluding thoughts
Contemporary views on mathematics learning and teaching have emphasized the need for learning and teaching mathematics through discourse. Research suggests that students be allowed to actively engage in generating mathematical knowledge by taking part in mathematical discussions and conversations. Such learning may be understood as a social collective practice from which the whole class as a collective benefits. This notion may help mathematics researchers, teachers and students to understand a mathematics classroom as a complex, self-updating autopoiesis system.

References


For the Learning of Mathematics, 15 (2).


[1] Here, I am using the word “discourse” to mean any “spoken or written treatment of a subject, in which it is handled or discussed at length” (Oxford English Dictionary). In this sense for me, discourse includes both discussions and conversations. For detailed distinctions between discussion and conversation, see Davis, 1996; Gordon Calvert, 2001.


[3] I am using “constructivism” in its general terms to include Radical and Social Constructivism.

[4] Another term that these authors use to describe their theory of learning, especially in recent work, is legitimate peripheral participation, which for several reasons I will choose not to use in this article.

About the Author

Helena Miranda is a graduate student in Secondary Education at the University of Alberta. She is interested in understanding better the dynamics of mathematics classroom discourses, with specific focus on how those discourses arise and are taken up for conversations and/or discussions.